# MODAL ANALYSIS OF A COAXIAL-LINE WAVEGUIDE JUNCTION

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## ABSTRACT

A modal approach is described for analysis of the junction between a coaxial line and a guiding structure having conducting planes on upper and lower surfaces. The expressions are verified by comparison with experimental measurements for the special case of a rectangular waveguide.

## INTRODUCTION

This paper sets out a technique for the analysis of the junction between a coaxial line and a wave-guiding structure. Its principal feature is that is applicable to guiding structures of interest at millimeter-wave frequencies, such as shielded image guide, H-guide and groove guide, as well as to conventional rectangular waveguide. It can also be applied to the case of a microstrip in place of a coaxial line, if the assumption of quasi-TEM mode propagation in the microstrip is accurate. The principal restriction on the waveguide is that it has perfectly-conducting planes on its upper and lower surfaces.

The basic elements of the analytical procedure are presented in this paper. The approach is then applied to the special case of the coaxial line and rectangular waveguide junction, since experimental measurements are readily available for this structure. Very good agreement is obtained between the measurements and theoretical values of impedance at the coaxial aperture plane.

## PREVIOUS WORK

Analysis has hitherto been confined to the special case when the guiding structure is a rectangular waveguide. The significant approaches have been:

(1) Lewin [1] replaced the coaxial line junction by a $\delta$-function voltage and a coaxial line characteristic impedance at the base of the post across the waveguide. This approach gave problems with convergence and the results do not agree well with measurements.

(2) Eisenhart *et al* [2] replaced the coaxial line by an equivalent gap in the post; this permits use of waveguide cylindrical post mount theory [3], [4]. The limitation here is that the equivalent gap size is determined by an empirical factor dependent on coaxial-line dimensions and impedance.

(3) Williamson [5] applied image theory to this problem, using the Love equivalence theorem to model the coaxial aperture by a magnetic surface current. His approach gives good agreement with theoretical results. However his method appears limited to rectangular waveguide, and is not readily extended to other forms of waveguide.

## METHOD OF ANALYSIS

We consider the general junction shown in Fig. 1(a) with a coaxial line intersecting a guiding structure. The analysis developed here characterizes this junction by determining the junction impedance (as seen from the coaxial aperture) and the current distribution on the inner conductor of the coaxial line.

The analysis commences with the assumption that the radial electric field in the aperture of the coaxial-line/waveguide intersection is a TEM mode component. Using the Love equivalence principle, the fields $\overline{E}_o$, $\overline{H}_o$ in the waveguide arising from coaxial-line excitation are identical to those produced by an equivalent magnetic current $\overline{M}$ flowing on the coaxial-line aperture surface $S_a$; as Harrington points out [6], this surface may have a perfect conductor just behind $\overline{M}$. The magnetic current is given by:

$$\overline{M} = \overline{n} \times \overline{E}_a \qquad (1)$$

where $\overline{n}$ is the unit normal to the aperture, and $\overline{E}_a$ is the TEM-mode electric field in the aperture. Hence we obtain:

$$\overline{M} = \overline{a}_\Phi \frac{V}{\ln(b/a)} \left(\frac{1}{r}\right) \qquad (2)$$

where V is the applied voltage, a and b are coaxial line dimensions, and a cylindrical co-ordinate system r, $\Phi$, y is used.

We now consider the coaxial-line outer conductor to be extended into the waveguide (the effect of these fictitious coaxial-line walls in the waveguide will be compensated for, see eqn.(4)). This extension creates a coaxial cavity which is excited by the source $\overline{M}$, as shown in Fig. 1(b), giving TEM mode fields $\overline{E}_1$, $\overline{H}_1$ and a current $I_1$ flowing on the inner conductor surface $S_1$. This current $I_1$ is readily found to be:

$$I_1 = -\frac{V}{Z_c} \frac{j \cos k(y-B)}{\sin kB} \qquad (3)$$

where $Z_c$ is the characteristic impedance of the coaxial line, B is the waveguide height, and k is the wave number.

There will also be a current $-I_1$ flowing on the inner surface $S_2$ of the outer coaxial cavity shown in Fig. 1(b).

We must now remove the outer-conductor walls shown in Fig. 1(b), to recover the original structure shown in Fig. 1(a). This removal is readily accomplished by use of a Schelkunoff field equivalence principle [7], through which a current $I_1$ on $S_2$ is used as a source giving fields $\overline{E}_2$, $\overline{H}_2$ in the waveguide as shown in Fig. 1(c). Note that this current $I_1$ on $S_2$ induces a current $I_2$ on the surface $S_1$. The desired field $\overline{E}_o$, $\overline{H}_o$ of Fig. 1(a) is given by the superposition of $\overline{E}_1$, $\overline{H}_1$ and $\overline{E}_2$ $\overline{H}_2$.

$E_{2y}$, the y-directed component of $\overline{E}_2$ is readily found in terms of the currents $I_1$(on $S_2$) and $I_2$(on $S_1$), if the modes in the general waveguide by specified by their field components $\overline{E}_{mn}$, $\overline{H}_{mn}$, and corresponding propagation

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constants $\Gamma_{mn}$, through:

$$E_{2y}(x,y,z) = -\sum_m\sum_n \left[ \frac{\int_{S_1+S_2} J_y(x',y',z') E_{ymn}(x',y') \, e^{-\Gamma_{mn}|z-z'|} \, dS'}{2\int_{S_1+S_2} [\bar{E}_{mn}(x',y') \times \bar{H}_{mn}(x',y')] \cdot \bar{a}_z \, dS'} \right] E_{ymn}(x,y) \qquad (4)$$

where $J_y$ represents the current density of $I_1$ and $I_2$ and the integrals are taken over $S_1$ and $S_2$ and the waveguide cross-sections. If $J_y$ were a delta function, (4) would represent one component of the dyadic Green's function for the general waveguide [7]. Note that (4) includes the current $I_2$ which has not yet been determined. This is accomplished by using an expansion based upon the magnetic field between conductors at y=0 and y=B:

$$I_2 = \sum_{n=0}^{\infty} \frac{A_n \varepsilon_{on}}{B} \cos k_{yn} y \qquad (5)$$

with $k_{yn} = \frac{n\pi}{B}$, and $\varepsilon_{on}$ is the Neumann factor. This $I_2$ form is used in (4), and the coefficients $A_n$ determined from the resulting $E_{2y}$ expression by imposing the condition:

$$\int_0^B E_{2y} \cos k_{yn} y \, dy = 0 \text{ for } n=0,\ 1,\ 2.. \qquad (6)$$

at r=a, i.e. the tangential component of electric field for each mode is zero at the perfectly-conducting surface $S_1$.

From the form of the excitation, it is seen that the $\bar{H}_2$ field can be represented as the sum of a set of radial-line TM modes (which may well have angular dependence). From radial-line theory for TM modes:

$$\int_a^b H_{2\Phi n} (r,\Phi) \, dr = \frac{jk}{Z_o(k^2 - k_{yn}^2)} \, E_{2yn} \Big|_{r=a}^{r=b} \qquad (7)$$

where $Z_o$ is the free-space wave impedance. Note that $E_{2yn}=0$ at r=a, in the waveguide of Fig. 1(c).

The admittance of the junction from the coaxial-line terminals at the aperture is given by [5]:

$$Y = \frac{\int_{S_a} (\bar{E}_o \times \bar{H}_o) \cdot \bar{a}_y \, dS}{V^2} \qquad (8)$$

From the components of $\bar{E}_o$ and $\bar{H}_o$, we obtain:

$$Y = -j\frac{\cot kB}{Z_c} + \frac{1}{V \ln(b/a)} \int_0^{2\pi}\int_a^b H_{2\Phi}(r,\Phi,y=0)\,dr\,d\Phi \qquad (9)$$

and the integral is readily evaluated using (7).

For the special case of small apertures an approximate expression for Y can be derived. From expression of the Maxwell equations in cylindrical co-ordinates, (9) can be written as:

$$Y = -j\frac{\cot kB}{Z_c} + \frac{I_2(y=0)}{V} + \frac{j\omega\varepsilon}{V\ln(b/a)} \int_0^{2\pi}\int_a^b r\ln\left(\frac{b}{r}\right) E_{2y}(r,\Phi,y=0)\,dr\,d\Phi \qquad (10)$$

This equation can be evaluated by expanding $I_2$ and $E_{2y}$ into Fourier components, as in (5). This expansion shows that for small apertures the integral term in (10) has a negligible effect, due to ln(b/r) = 0 at r=b and $E_{2y}$ = 0 at r=a, and accurate results are obtained with this term neglected.

## RESULTS

The analysis set out above is applicable to all guiding structures having perfectly-conducting planes on upper and lower surfaces. It is here applied to the special case of the rectangular waveguide, for the purpose of experimental verification using published measurements [2][5]. The results are shown in Figs. 2-4, for standard X-band and S-band waveguide, with coaxial line of 50Ω, 66Ω and 24.5Ω characteristic impedance with a total of $M = \frac{3}{2}\frac{A}{a}$ modes considered in the x direction and a small number of modes in the y direction.

From (4) and (7), it is clear that the input impedance values will depend on the value of Φ at which the field $H_{2\Phi}$ is evaluated. Calculations show that the current $I_2$ varies with Φ, to an extent proportional to the ratio of the post diameter and the waveguide width; thus, this variation is slight for thin posts.

Figs. 2 and 4 show good agreement between theory and measurement for small and medium sized apertures, using Φ= π/2, while Fig. 3 shows that for a large aperture. Fig. 4 also shows that the approximate formula in (10) gives good accuracy.

## CONCLUSIONS

The theory developed here has been shown to give good accuracy, through comparison with experimental measurements. It will probably find its most significant application in millimeter-wave circuit design, with dielectric waveguides or H-guide structures.

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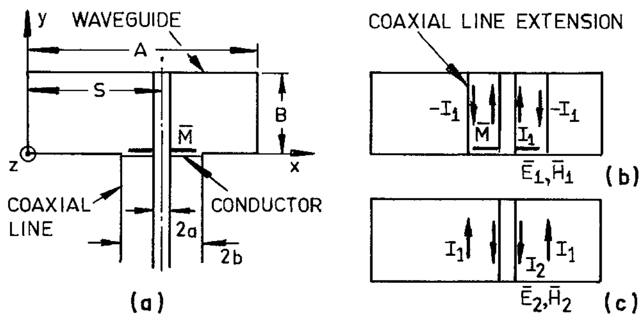


Fig. 1(a) Coaxial line-waveguide junction:
(b) Extended coaxial line structure used to find one admittance component; (c) current sources compensating for Fig. 1(b) currents, so that (b) and (c) combine to represent (a).

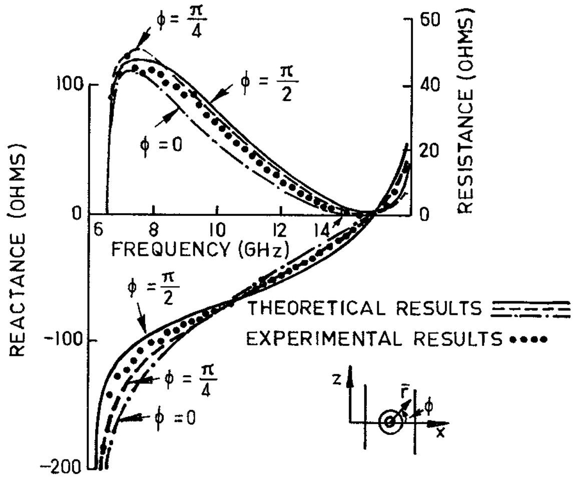


Fig. 2 Comparison between theoretical and experimental values of input impedance at the coaxial aperture plane, for A=22.86mm, B=10.16mm, S=A/2, and with a=1.55mm, b=3.55mm for 50Ω line.

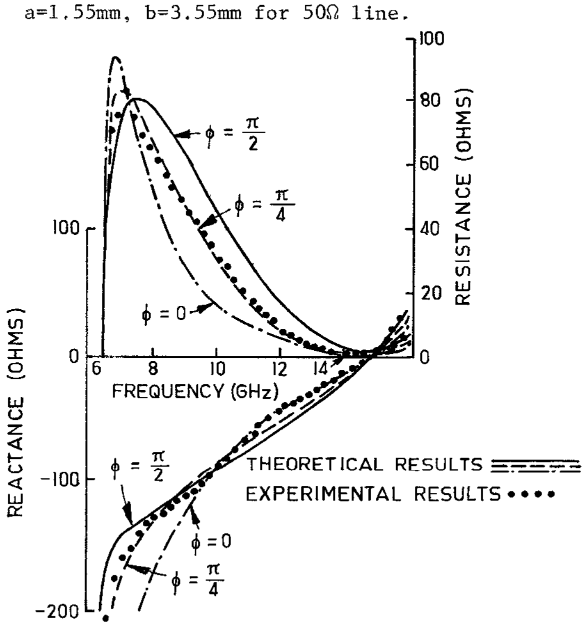


Fig. 3 As for Fig. 2, but with a=2.37mm, b=7.15mm, for 66Ω line.

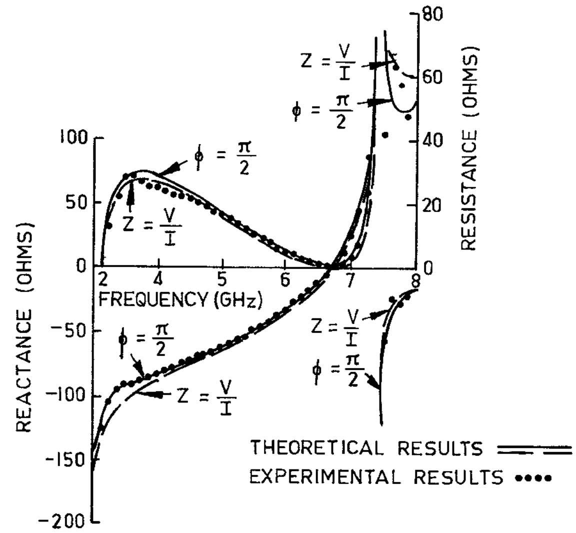


Fig. 4 As for Fig. 2, but with A=47.6mm, B=22.15mm. S=A/2, and a=2.37mm, b=3.55mm, for 24.5Ω line.